The Power Law: An Example of a Behavioral Illusion?

Richard S. Marken

Department of Psychology

University of California, Los Angeles

July 9, 2008

Correspondence regarding this paper should be directed to the author at:

1285 Franz Hall, Box 951563

Los Angeles, CA 90095-1563

310 729 - 1400

rsmarken@gmail.com

The Power Law: An Example of a Behavioral Illusion?

Richard S. Marken

Abstract: Perceptual Control Theory (PCT) shows that a "behavioral illusion" can occur when studying closed-loop control systems. The illusion is that an observed relationship between environmental inputs and behavioral outputs reflect characteristics of the system itself when it actually reflects properties of the feedback connection between the system's output and a controlled perceptual input. A possible example of such an illusion is Stevens' Power Law, which says that the observed relationship between input and output in magnitude estimation experiments represents perceptual characteristics of the person under study. A PCT analysis shows that the Power Law is more likely to represent the inverse of a logarithmic feedback function connecting magnitude estimates to the perceived difference between numerical and stimulus magnitude. The possible existence of this behavioral illusion suggests that the first step in psychological research should be to test whether the behavior under study is that of a closed-loop system.

According to the Perceptual Control Theory (PCT) model of behavioral organization, living organisms are closed-loop systems that act to keep perceptual variables in pre-specified states, protected from disturbances caused by variations in environmental circumstances (Powers, 1973b). This process is called *control* and the perceptual variables that are maintained in pre-specified states are called *controlled variables* (Marken, 2001, page 101). To an outside observer, the actions that protect a controlled variable from disturbance will appear to be outputs that are caused by those disturbances. This is especially true if the existence of the controlled variable itself goes unnoticed (Cziko, 2000, page 88).

The causal path from disturbance to output will appear to run in one direction, starting with the disturbance, going through the behaving system and ending with the output action. The relationship between disturbance and output variations will, therefore, appear to represent the transfer function that characterizes the internal organization of the behaving system. However, in a closed-loop control system this relationship actually reflects characteristics of the feedback connection between system output and controlled variables.

Thus, when one deals with a closed-loop control system, observed relationships between environmental disturbance and behavioral output variables can create a "behavioral illusion" (Powers 1978, page 421). The illusion is that the relationship between environmental and behavioral variables reflects characteristics of the system under study when, in fact, it reflects characteristics of the environmental feedback function that connects system output to a controlled perceptual input: the controlled variable.

An Example of a Behavioral Illusion

An example of a behavioral illusion can be seen in a simple tracking type experiment. Suppose that a participant is asked to operate a control lever to place a pointer next to a vertically movable target. As the target is moved up and down, from one position to another, the participant moves the lever to bring the pointer to each new position. For simplicity we will consider only one-dimensional tracking. If the target is moved 10 cm in one direction it is observed that the participant moves the hand on the lever through an arc of, say, 20 degrees in the opposite direction, and this relationship holds for all movements of the target. So it seems that there is a simple cause-effect relationship such that each centimeter of target movement is sensed by the participant, and causes the hand to move 2 degrees.

Now let us say that behind the scenes, the effect of the lever on the pointer is doubled, by moving the pivot point closer to the place where the lever is grasped. If the participant continues to succeed in bringing the pointer to the target position each time the target moves, we will now see that a 10-cm movement of the target produces a hand movement of only 10 degrees. It now seems that the sensitivity of the participant to target movements has changed: a stimulus of 10 cm of target movement causes only half as much response as before. It seems as if the participant has become less responsive to changes in the stimulus.

In fact, the responsiveness of the participant has not changed at all. The task is to put the pointer next to the target, and this is accomplished whether doing it requires a large or a small movement

of the lever. The ratio of hand movement to target movement is determined, in that case, entirely

by the placement of the fulcrum, which changes the "feedback function" that connects the

participant's action (lever movement) to the position of the pointer. The feedback function is a

simple constant of proportionality in this thought experiment, but the same principle will

obviously hold for any form of the feedback function, given that the participant continues to keep

the pointer on the target. The lever angle, and hence the hand position, is just the inverse

feedback function of the target position -- a property of the physical environment, not of the

participant.

The Behavioral Illusion and Scientific Psychology

Powers (1973a) has pointed out that the possible existence of this behavioral illusion has

important implications for scientific psychology. Most experimental research in psychology

involves manipulation of an environmental stimulus variable, e, as an independent variable and

measurement of concomitant variation in a behavioral response variable, o, as a dependent

variable (Marken, 1997). Any observed functional relationship between e and o is assumed to

reflect characteristics of the organism under study. If o is found to be a function of e, such that o

= f(e), the function, f, is assumed to reflect characteristics of the organism.

If, however, the organism under study is a closed-loop control system then e is likely to be a

disturbance to a perceptual variable, p, that the organism is controlling and o is likely to be the

means the organism uses to counter the effects of e and keep p under control. That is, the

independent variable in an experiment is likely to be equivalent to the moving target in the

tracking example described above. The dependent variable is likely to be equivalent to the

movement of the hand on the lever.   And the controlled variable, p, would be equivalent to the

distance between pointer and target.   The controlled variable, if it exists in an experiment, will

be difficult to notice because it is a perception controlled by the organism itself, not by the

experimenter.

An observed functional relationship between independent and dependent variable in a

psychology experiment is assumed to reflect characteristics of the organism under study. Thus, if

o is found to be a function of e, such that $o = f(e)$, the function, f, is assumed to reflect

characteristics of the organism. If, however, the organism is a closed-loop control system, then f

is actually the inverse of the feedback function, g, which relates o to the controlled variable, p.

That is, if $p = g(o)$ then $f = g^{-1}$.

So the functional relationship between independent and dependent variable found in an

experiment on a closed-loop system will appear to reflect properties of the organism when, in

fact, it may actually reflect properties of the environmental feedback connection between an the

organism's output and a perceptual variable it is trying to keep under control. Thus, the observed

functional relationship between e and o that seems to characterize the organism under study may

be a behavioral illusion if the organism is actually a closed-loop control system.

Stevens' Power Law

It is somewhat difficult to find possible examples of the behavioral illusion in the psychological research literature because the environmental variables manipulated in psychological experiments are typically qualitative. Therefore, the results of such experiments are rarely expressed in the form of an equation showing the quantitative relationship between the independent (e) and dependent (o) variable. One example of research where the relationship between e and o is expressed in the form of an equation is in studies of the relationship between stimulus and psychological magnitude. These are the magnitude estimation experiments developed by S. S. Stevens (1957).

In magnitude estimation experiments a "standard" stimulus is presented and the participant is told a value to assign to it: a "modulus". Then the same stimulus with a different magnitude is presented and the participant is told to express its perceived magnitude as a second number. The ratio of the second number to the modulus is taken to be the ratio of perceived magnitudes of the stimuli. The obtained data led to the Power Law, where the exponent and the constant of proportionality were determined by curve-fitting:

(1) $o_n = k e_s^{a}$

where e_s is the environmental stimulus variable and o_n is the participant's numerical response,

which is taken to be proportional to the perceptual magnitude of e_s, $\psi(e_s)$. Assuming as Stevens did that $\psi(e_s) = o_n$, we can write

(2) $\psi(e_s) = k_S \, e_s^a$

The Power Law caused quite a sensation because it seemed to contradict Fechner's law, which held that the relationship between stimulus and perceptual magnitude is logarithmic (Boring, 1950). Fechner's law is given by the following equation:

(3) $\psi(e_s) = k_F \log(e_s)$.

Fechner's law was based on Ernst Weber's finding that the size of a Just Noticeable Difference or JND between two magnitudes of the same stimulus, e_{s1} and e_{s2}, was proportional to stimulus magnitude. This translates into equation 1, assuming that the subjective size of the JND, in terms of the perceptual magnitudes of e_s, $\psi(e_{s1})$ and $\psi(e_{s2})$, is constant over all magnitudes of e_s.

Control of Magnitude Estimates

The two laws can be reconciled by recognizing that the Power Law may be an example of a behavioral illusion. The exponential relationship between e and o that is observed in magnitude estimation experiments seems to characterize the nature of the participant's response of different

stimulus magnitudes but it may actually characterize the nature of the feedback function in a control loop. This possibility can seen by recognizing that the participant in a magnitude estimation task is *controlling* a relationship between the perception of stimulus magnitude and a perception of the number given as an estimate of that magnitude.

The situation is shown in Figure 1. The independent variable in the experiment is the magnitude of the stimulus presented on each trial, e_s, and the dependent variable is the number assigned to that stimulus, o_n. The participant is to produce a value of o_n that somehow matches the magnitude of e_s. Interpreted as a control process, this means that the participant must keep the perception of o_n, $\psi(o_n)$, equal to the perception of e_s, $\psi(e_s)$, on each trial. So the controlled perception in the experiment is $\psi(e_s) - \psi(o_n)$ and the participant is to keep this difference equal to zero (as indicated by the 0 next to the reference in Figure 1), which will happen when $\psi(e_s) = \psi(o_n)$.

Figure 1 Here

In order to keep $\psi(e_s) - \psi(o_n) = 0$ on each trial the participant must produce numerical estimates, o_n, that are perceived to be equal to the perceived stimulus magnitudes, $\psi(e_s)$. The relationship between numerical estimates, o_n, and how they are perceived, $\psi(o_n)$, is described by the function connecting o_n to $\psi(o_n)$ in Figure 1. This is a feedback function but in this case the function is a characteristic of the perceptual rather than the environmental connection between output and

input. This is the function that relates the perception , $\psi(o_n)$, of the magnitude of numbers to the number stimuli themselves, o_n. Assuming that Fechner's law applies to the perception of the magnitude of numbers as well as to the magnitude of any other stimulus, we assume that

(4) $\psi(o_n) = k_1 \log (o_n)$

for the perception of number magnitude and

(5) $\psi(e_s) = k_2 \log (e_s)$

for the perception of stimulus magnitude.

When the participant in a magnitude estimation experiment selects a number, o_n, such that $\psi(o_n) = \psi(e_s)$ (keeping the controlled perception $\psi(o_n) - \psi(e_s) = 0$) it follows that:

(6) $k_1 \log(o_n) = k_2 \log(e_s)$, or after simplification,

(7) $o_n = e_s^{(k2/k1)}$

Thus, assuming that participants are controlling a relationship between perceptions of stimulus and numerical magnitude in a magnitude estimation task and that Fechner's law applies to

perception of the sizes of numbers as well as other stimuli, leads to predicting that in the magnitude estimation experiment the participant will pick a number proportional to a power of the stimulus.

Equation (7) is an example of the behavioral illusion to the extent that it is seen as a reflection of the nature of the human response to varying stimulus magnitudes. Control theory shows that the the Power Law relationship described by equation (7) is not a characteristic of the organism's perceptual system but, rather, is the inverse of the logarithmic feedback relationship that exists between numerical outputs, o_n, and the controlled perception: $\psi(o_n)$ - $\psi(e_s)$.

Discussion

A PCT analysis of the magnitude estimation task (Figure 1) shows that the observed Power Law relationship between the size of stimuli, e_s, and the numbers that are used to describe their size, o_n, could be a behavioral illusion in the sense that it could be the inverse of a logarithmic feedback connection between actions and a controlled perceptual variable. Participants in a magnitude estimation experiment can be seen to be controlling the relationship between the perception of e_s and o_n; this relationship is a controlled perceptual variable. The logarithmic function relating numerical responses to the perception of those responses represents the feedback connection between the participants' outputs and the controlled variable. PCT predicts that the relationship between e_s and o_n will be the inverse of this feedback function, which is the

11

Power Law. When magnitude estimation is seen as the control of the relationship between $\psi(o_n)$

and $\psi(e_s)$, Stevens' Power Law can be seen as a behavioral illusion completely predicted by

Fechner's logarithmic law. The Power Law describes not characteristics of the perceptual system

of the organism but the inverse of those characteristics.

The inverse relationship between Stevens' Power Law and Fechner's Logarithmic Law has been

known for some time (MacKay, 1963).   What is original in the present analysis is the use of this

relationship to illustrate a "behavioral illusion" that can occur in research aimed at determining

the internal organization of what turns out to be a closed-loop control system. In the magnitude

estimation experiment, the "illusion" is that the internal organization (perceptual function) that

relates stimulus to response magnitude is the observed Power Law when, in fact, it is a

logarithmic function. This illusion occurs because magnitude estimation is a closed-loop task

that requires control of the perception of a relationship between two stimulus magnitudes.

The possible existence this "behavioral illusion" should serve as a caution to all researchers who

study behavior by looking at the relationship between environmental variables and behavioral

responses. An "illusory" relationship between these variables will be seen whenever an

environmental variable (such as stimulus magnitude, e_s in the magnitude estimation task) acts as

a disturbance to a perceptual variable (such as the relationship between stimulus magnitudes)

that is being controlled by a closed-loop control system. The control system will act to counter

the disturbance with behavior (such as the numerical response, o_n, in the magnitude estimation

task) that operates on the controlled variable through an environmental feedback path. This creates the illusion that the observed relationship between environmental and behavioral variables describes a causal path through the organism when, in fact, it describes the inverse of the feedback connection between behavior and controlled variable (Powers, 1973a, 1978).

The way to avoid the behavioral illusion is to make sure that there is no possibility that the system under study is closed-loop with respect to the variables in an experiment. This can be done using the control theory-based Test for the Controlled Variable or TCV (Marken, 1997; 2001; 2005). The TCV makes it possible to determine whether the independent and dependent variables can be treated as the beginning and end on an open-loop causal chain (as is typically assumed) or must be treated as having opposing influences on a perceptual variable controlled as part of a closed-loop.

Acknowledgment

References

Boring, E. G. (1950). *History of experimental psychology* (pp. 27-49). New York: Appleton-Century-Crofts.

Cziko, G. (2000) *The things we do: Using the lessons of Bernard and Darwin to understand the what, ho and why of our behavior.* Cambridge, MA: MIT Press

MacKay, D.M. (1963). Psychophysics of perceived intensity: A theoretical basis for Fechner's and Stevens' laws. *Science*, 139, 1213-1216.

Marken, R. S. (1997) The Dancer and the Dance: Methods in the Study of Living Control Systems, *Psychological Methods*, 2 (4), 436-446

Marken, R. S. (2001) Controlled variables: Psychology as the center fielder views it, *American Journal of Psychology, 114(2)*, 259-282

Marken, R. S. (2005) Optical Trajectories and the Informational Basis of Fly Ball Catching, *Journal of Experimental Psychology: Human Perception & Performance,* 31 (3), 630 – 634

Powers, W. T. (1973a) Feedback: Beyond behaviorism. *Science*, 179, 351-356

Powers, William T. (1973b). *Behavior: The control of perception*. Chicago: Aldine

Powers, W. T. (1978). Quantitative analysis of purposive systems: Some spadework at the foundations of scientific psychology. *Psychological Review*, 85, 417-435

Stevens, S. S. (1957). On the psychophysical law. *Psychological Review* 64(3):153–181.

Figure Caption
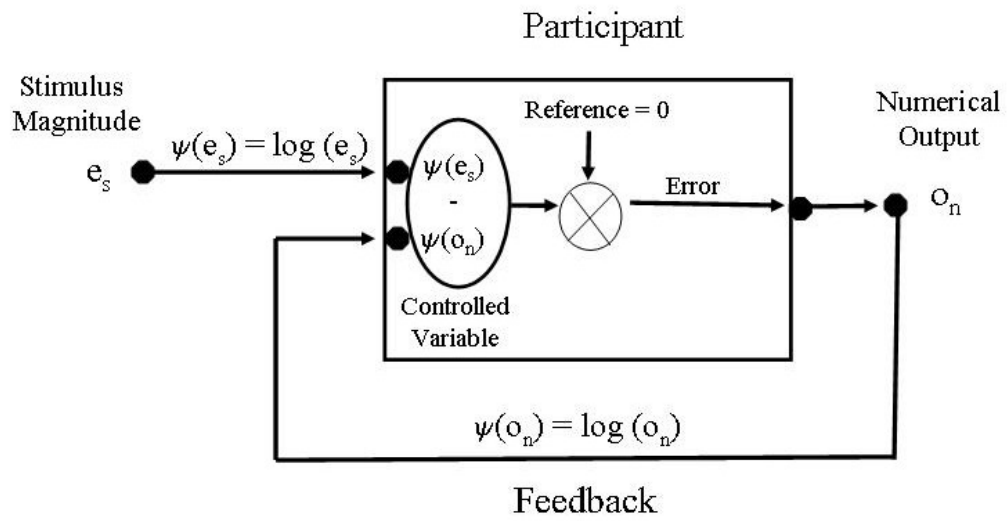

Figure 1. Closed-loop analysis of magnitude

Participant

Stimulus
Magnitude

$\psi(e_s) = \log(e_s)$

e_s

Reference = 0

$\psi(e_s)$
-
$\psi(o_n)$

Error

Numerical
Output

o_n

Controlled
Variable

$\psi(o_n) = \log(o_n)$

Feedback

Figure 1